Title: Graphing Functions – Translations and Reflections
Class: Math 107 or Math 111
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Instructions to tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: Graph, translation, reflection, shift

Graphing Functions – Translations and Reflections

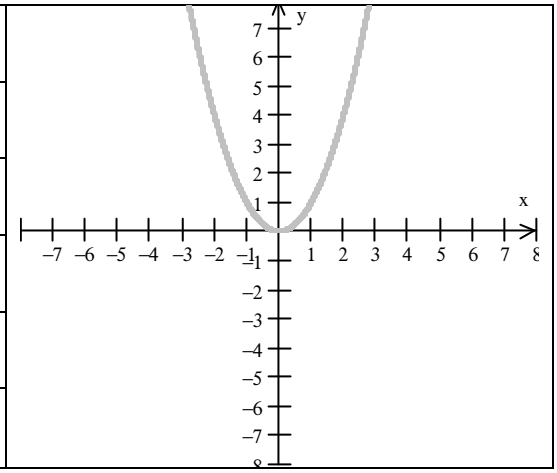
Purpose:
This is intended to refresh your knowledge about graphing functions, including translations and reflections of graphs.

First we will take a look at the function $y = x^2$ and various horizontal and vertical shifts or *translations*.

Example: Complete the following tables of values and sketch the graph of each function. The graph of $y = x^2$ is shown for reference.

(a) $f(x) = x^2 + 2$

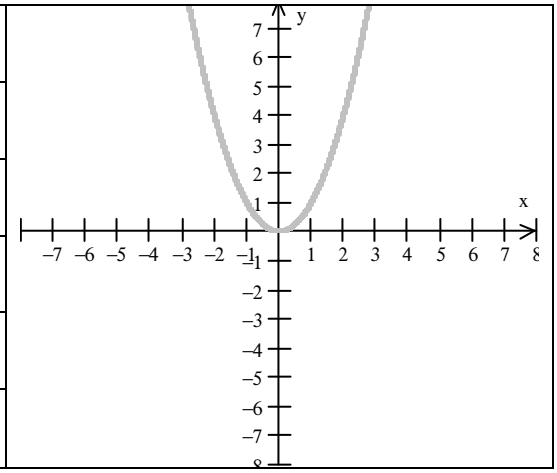
x	$f(x)$
-2	
-1	
0	
1	
2	



What was the effect on the original function $y = x^2$?

(b) $f(x) = x^2 - 2$

x	$f(x)$
-2	
-1	
0	
1	
2	

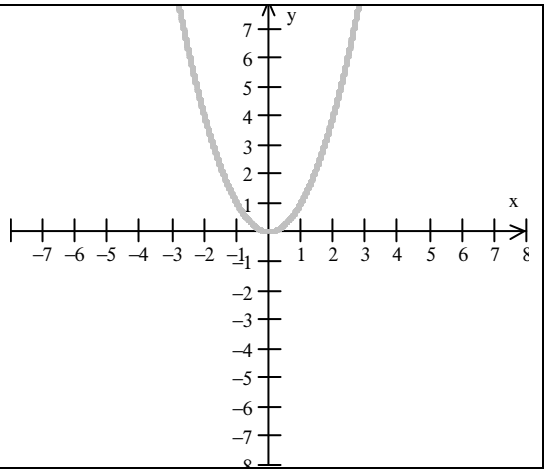


What was the effect on the original function $y = x^2$?

Example: Complete the following tables of values and sketch the graph of each function.

(a) $f(x) = (x+2)^2$

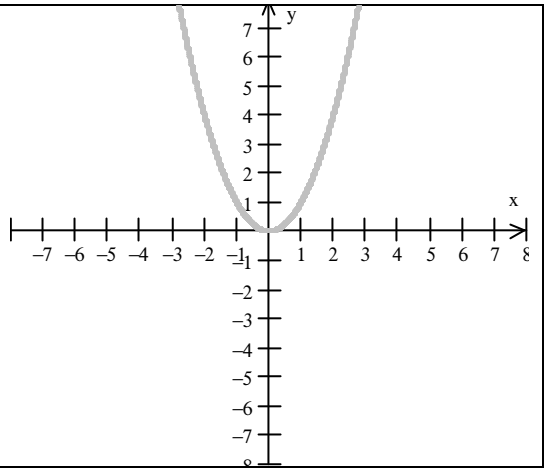
x	$f(x)$
-2	
-1	
0	
1	
2	



What was the effect on the original function $y = x^2$?

(b) $f(x) = (x-2)^2$

x	$f(x)$
-2	
-1	
0	
1	
2	



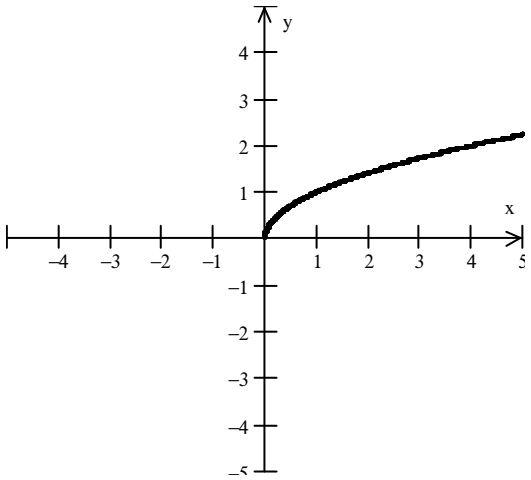
What was the effect on the original function $y = x^2$?

Vertical and Horizontal Translations Summary – Suppose f is a function and c is a positive real number.

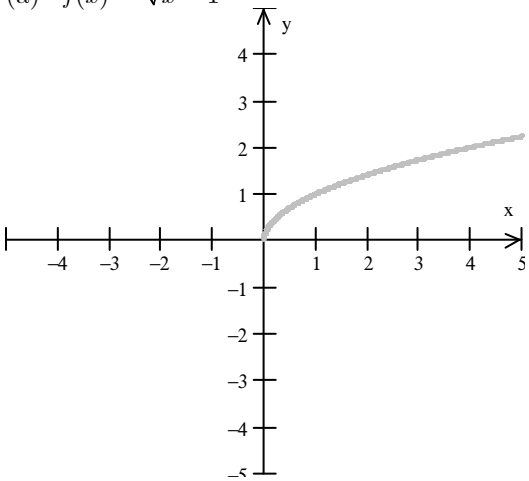
- $f(x)+c$ shifts the graph of $y = f(x)$ up c units
- $f(x)-c$ shifts the graph of $y = f(x)$ down c units
- $f(x+c)$ shifts the graph of $y = f(x)$ to the left c units
- $f(x-c)$ shifts the graph of $y = f(x)$ to the right c units

Try the next couple problems on your own, without making a table of values.

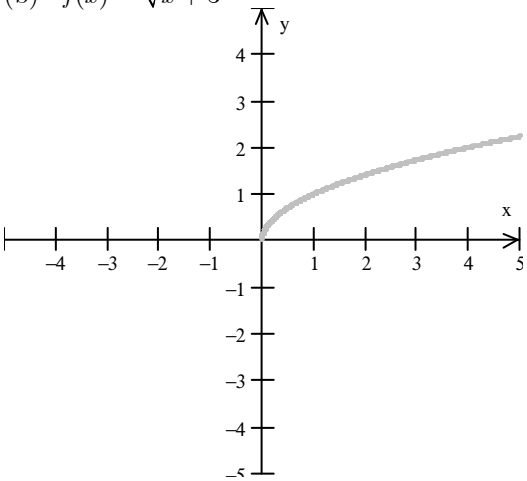
1. Consider the function $y = \sqrt{x}$. Use the ideas of horizontal and vertical translation to sketch each function – do these without making a table of values, rather, shift the original function appropriately.



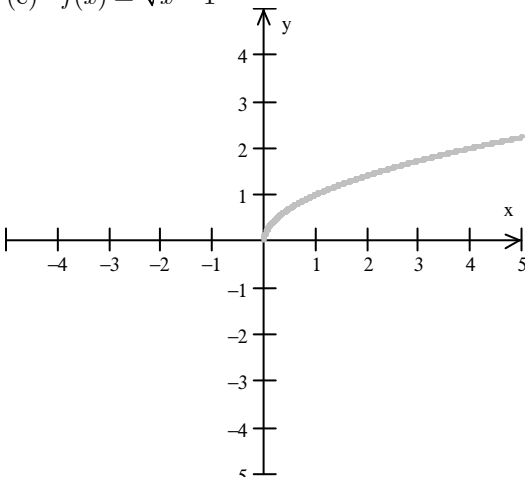
(a) $f(x) = \sqrt{x} - 1$



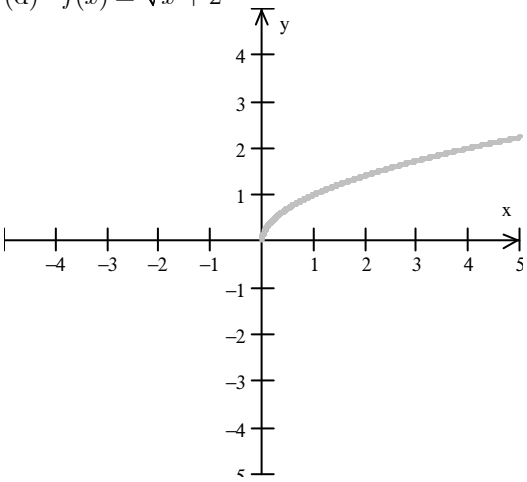
(b) $f(x) = \sqrt{x + 3}$



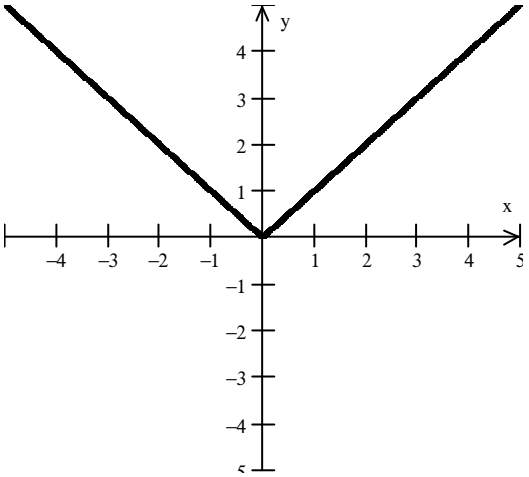
(c) $f(x) = \sqrt{x - 1}$



(d) $f(x) = \sqrt{x} + 2$

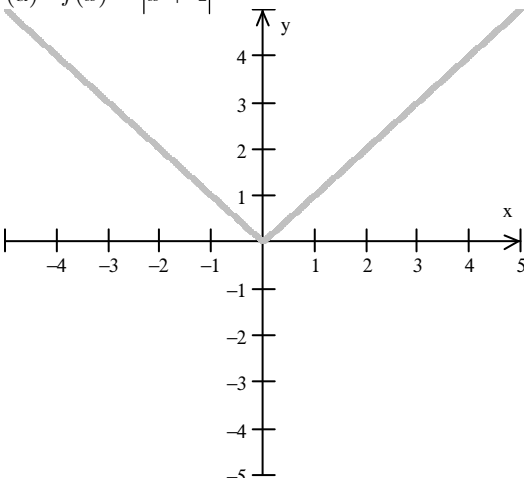


2. Consider the function $y = |x|$. Use the ideas of horizontal and vertical translation to sketch each function – do these without making a table of values, rather, shift the original function appropriately.

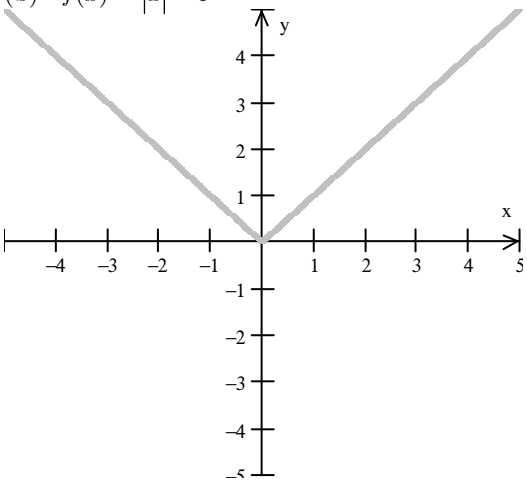


$y = |x|$

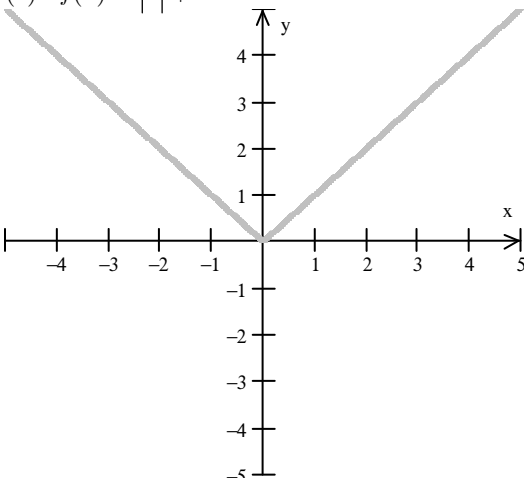
(a) $f(x) = |x + 4|$



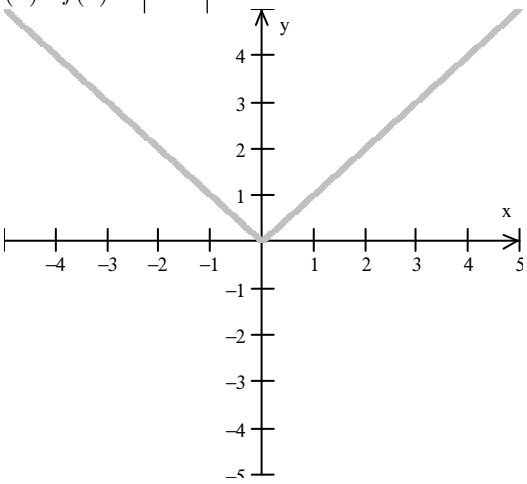
(b) $f(x) = |x| - 5$



(c) $f(x) = |x| + 1$



(d) $f(x) = |x - 2|$

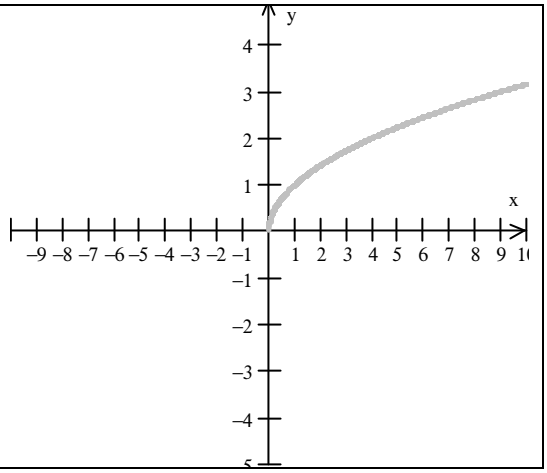


Now that you understand vertical and horizontal translations, let's take a look at another graphing transformation, called a *reflection*.

Example: Complete the following tables of values and sketch the graph of each function.

(a) $f(x) = -\sqrt{x}$

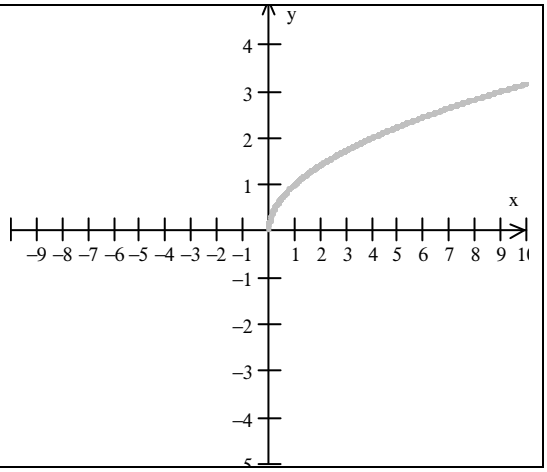
x	$f(x)$
0	
1	
4	
9	



What was the effect on the original function $y = \sqrt{x}$?

(b) $f(x) = \sqrt{-x}$

x	$f(x)$
0	
-1	
-4	
-9	



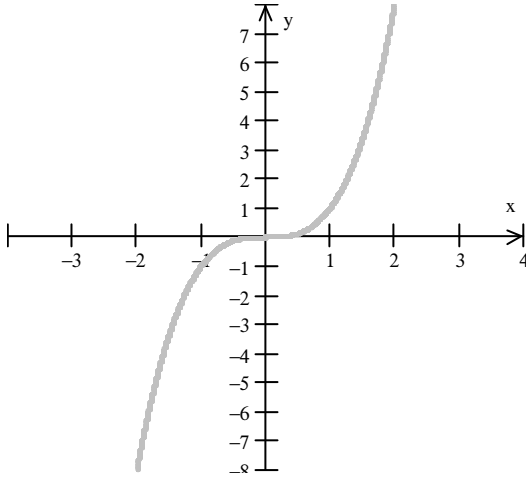
What was the effect on the original function $y = \sqrt{x}$?

Reflection Summary – Suppose f is a function.

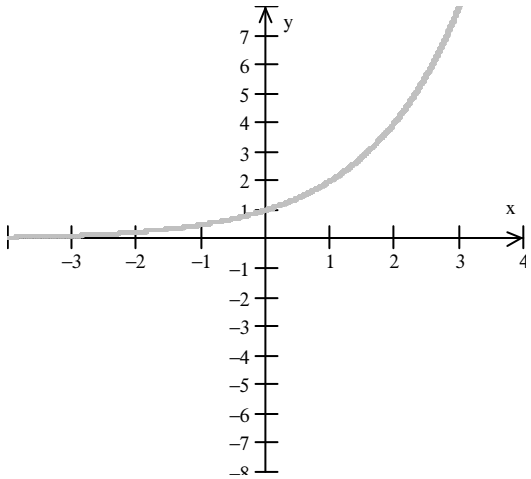
- $-f(x)$ reflects the graph of $y = f(x)$ about the x-axis
- $f(-x)$ reflects the graph of $y = f(x)$ about the y-axis

Try the next couple problems on your own, without making a table of values.

3. The graph of the function $y = x^3$ is shown. Use it to sketch $f(x) = -x^3$ on the same set of axes.



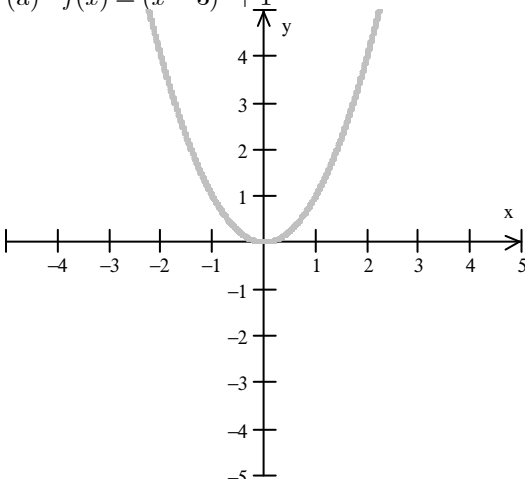
4. The graph of the function $y = 2^x$ is shown. Use it to sketch $f(x) = 2^{-x}$ on the same set of axes.



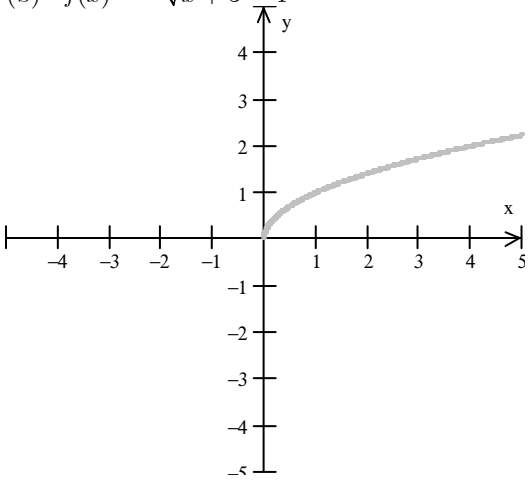
The remaining problems combine all of the ideas illustrated above. One important thing to note – do any reflection first, and then apply a shift. Check your results on your graphing calculator.

5. Sketch each of the following – write down what each function does to the basic function to which it is related. When combining translations and reflections, always do the reflections first, then shift.

(a) $f(x) = (x-3)^2 + 1$



(b) $f(x) = -\sqrt{x+3} - 4$



(c) $f(x) = -(x+2)^3 - 1$