Title: Rational Equations, Level I.
Class: Math 100 or Math 107
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Instructions to Tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: solving rational equations, rational equations, equations with rational expressions

Rational Equations, Level I.

Purpose: This is intended to refresh your skills in solving rational equations.

Activity: Work through the following activity and examples. Do all of the practice problems before consulting with a tutor.

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To solve equations containing rational expressions, we are going to use a property of equations: multiply both sides of the equation by any non-zero expression, and the solutions to the old equation are contained in the solutions to the new equation.

Example 1 $\dfrac{3}{4}+\dfrac{5}{8}=\dfrac{x}{12}$ The LCD is 24. Multiply both sides of the equation by 24.

$$24\left(\dfrac{3}{4}+\dfrac{5}{8}\right)=24\left(\dfrac{x}{12}\right)$$

$$24\cdot\dfrac{3}{4}+24\cdot\dfrac{5}{8}=24\cdot\dfrac{x}{12}$$ Use the Distributive Property to remove the parentheses.

$$\dfrac{6\,\cancel{24}}{1}\cdot\dfrac{3}{\cancel{4}\,1}+\dfrac{3\,\cancel{24}}{1}\cdot\dfrac{5}{\cancel{8}\,1}=\dfrac{2\,\cancel{24}}{1}\cdot\dfrac{x}{\cancel{12}\,1}$$

$$6\cdot3+3\cdot5=2\cdot x$$
$$18+15=2x$$
$$33=2x$$
$$x=\dfrac{33}{2}$$

Practice 1a $\dfrac{3}{4}+\dfrac{7}{8}=\dfrac{x}{2}$ **Practice 1b** $\dfrac{2}{3}-\dfrac{4}{5}=\dfrac{x}{15}$

Did you get 1a) $\dfrac{13}{4}$ and 1b) -2 ? Very good!

Example 2 $\quad \dfrac{1}{2x} + \dfrac{1}{x} = -12$ $\qquad$ The LCD is 2x. The restrictions (values that make any

denominator equal to zero) are $x \neq 0$

$$\dfrac{\cancel{2x}}{1} \cdot \dfrac{1}{\cancel{2x}} + \dfrac{2\cancel{x}}{1} \cdot \dfrac{1}{\cancel{x}} = \dfrac{2x}{1} \cdot \dfrac{-12}{1}$$
$$1 + 2 = -24x$$
$$3 = -24x$$
$$x = \dfrac{3}{-24} = -\dfrac{1}{8}$$

Practice 2a $\quad \dfrac{2}{3x} + \dfrac{1}{x} = 10$ $\qquad\qquad\qquad$ **Practice 2b** $\quad \dfrac{5}{x} = \dfrac{6}{x} - 1$

Did you get 2a) $\dfrac{1}{6}$ and 2b) 1 ?

Example 3 $\quad x - \dfrac{6}{x} = -5x$ $\qquad$ LCD is x, and $x \neq 0$

$$\dfrac{x}{1} \cdot \dfrac{x}{1} - \dfrac{\cancel{x}}{1} \cdot \dfrac{6}{\cancel{x}} = \dfrac{x}{1} \cdot \dfrac{-5x}{1}$$
$$x^2 - 6 = -5x^2$$
$$6x^2 - 6 = 0$$
$$6\left(x^2 - 1\right) = 0$$
$$6(x-1)(x+1) = 0$$
$$x = 1 \text{ or } x = -1$$

Practice 3 $\quad x - \dfrac{4}{x} = -3x$

Did you get $x = 1$ or -1?

Example 4 $\dfrac{x-1}{x-5} = \dfrac{4}{x-5}$ LCD is $(x-5)$, and $x \neq 5$

$$\dfrac{\cancel{x-5}}{1} \cdot \dfrac{x-1}{\cancel{x-5}} = \dfrac{\cancel{x-5}}{1} \cdot \dfrac{4}{\cancel{x-5}}$$

$$x-1=4$$

$x = 5$, but $x = 5$ will make the denominators zero. So we have to reject $x = 5$.
No solution.

Practice 4 $\dfrac{x+5}{x+2} = \dfrac{3}{x+2}$

Did you get no solution also?

Example 5 $\dfrac{x^2}{x-1} = \dfrac{1}{x-1}$ LCD is $(x-1)$, and $x \neq 1$

$$\dfrac{x-1}{1} \cdot \dfrac{x^2}{x-1} = \dfrac{x-1}{1} \cdot \dfrac{1}{x-1}$$

$$x^2 = 1$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$x = 1$ or $x = -1$, but $x = 1$ will make the denominator zero, so we reject $x = 1$.
The solution is $x = -1$.

Practice 5 $\dfrac{x^2}{x+2} = \dfrac{4}{x+2}$

Did you get $x = 2$, rejecting $x = -2$ which would make the denominator zero?

Example 6 $\dfrac{1}{x+3}=\dfrac{5}{x}+4$ LCD is $x(x+3)$, and $x \neq -3$ or 0

$$\dfrac{x\cancel{(x+3)}}{1}\cdot\dfrac{1}{\cancel{x+3}}=\dfrac{\cancel{x}(x+3)}{1}\cdot\dfrac{5}{\cancel{x}}+\dfrac{x(x+3)}{1}\cdot\dfrac{4}{1}$$

$$x\cdot 1=(x+3)\cdot 5 + x(x+3)\cdot 4$$

$$x=5x+15+4x^2+12x$$

$$0=4x^2+16x+15$$

$$0=(2x+3)(2x+5)$$

$$x=-\dfrac{3}{2}\ \text{ or }\ -\dfrac{5}{2}$$

Practice 6 $\dfrac{2x}{x+1}=2-\dfrac{5}{2x}$

Did you get $x=-5$? Excellent. Now try some more on your own.

Problems

1. $\dfrac{5}{x}+\dfrac{1}{2}=1$

2. $\dfrac{1}{x}+\dfrac{1}{2x}=3$

3. $\dfrac{2}{x}+\dfrac{1}{3}=\dfrac{1}{2x}-1$

4. $\dfrac{x}{x-2}=\dfrac{4}{3}$

5. $\dfrac{x-7}{x+2}=\dfrac{1}{4}$

6. $\dfrac{1}{x}+\dfrac{2}{x}+\dfrac{3}{x}=2$

7. $\dfrac{x-7}{x-9}=\dfrac{2}{x-9}$

8. $\dfrac{2x}{x+1}=2-\dfrac{5}{2x}$

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Review: Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

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For Tutor Use: Please check the appropriate statement:

_____Student has completed worksheet but may need further assistance. Recommend a follow-up with the instructor.

_____Student has mastered topic.